On D3-brane Dynamics at Strong Warping

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Abstract

We study the dynamics of a D3 brane in generic IIB warped compactifications, using the Hamiltonian formulation discussed in [1]. Taking into account of both closed and open string fluctuations, we derive the warped Kähler potential governing the motion of a probe D3 brane. By including the backreaction of D3, we also comment on how the problem of defining a holomorphic gauge coupling on wrapped D7 branes in warped background can be resolved.

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1 Introduction

A recurrent theme in recent developments of string theory has been the utility of warped geometries in an increasingly wide variety of physics contexts. In additional to its pivotal role in understanding strongly coupled field theories via the gauge/gravity correspondence [2, 3, 4], warping has also become an indispensable tool in many constructions of particle physics and cosmology models from string theory. The gravitational redshift due to warping suggests a mechanism to generate a hierarchy of scales[1], thus realizing in a geometric way [5] the idea of technicolor. Even within the framework of supersymmetry, the omnipresent strongly coupled hidden sectors often admit gravity duals that are warped and as such provide a holographic description of supersymmetry breaking and its mediation (see, e.g., [10, 11]). Likewise, warping enables us to determine the structure of the inflation potential for some string inflationary scenarios [12] by utilitizing the systematics of the AdS/CFT duality [13], not to mention that it also introduces tunable small numbers in the inflaton potential useful for model building purposes.

Given the variety of applications, it is of interest to determine the low energy effective action governing string theory on strongly warped backgrounds. In particular, to draw precise predictions in models of particle physics and cosmology constructed in this framework, one needs to take into account all warping contributions to the effective action, including their effects on the Kähler potential which is not protected by holomorphy[2]. While the vacuum structure of warped compactifications have been well explored, issues involving the *dynamics* of such backgrounds are much less understood. In fact, the derivation of a warped effective theory describing fluctuations around a strongly warped background has proven to be highly subtle [14, 15, 16, 17]. Though significant progress has recently been made in computing the effective action for the closed string sector [1, 16] (and in some cases, simple expressions of the warped Kähler potential were presented [18]), the inclusion of open string degrees of freedom has so far been carried out [19] (see also [20]) only in the D-brane probe limit and in a fixed closed string background. By demanding the combined Kähler potential to reproduce the correct kinetic terms for the open string fluctuations in a warped background, one can constrain its form [19], and indeed the results find agreement with previous closed string computations. However, to go beyond these limited examples, it is important to extend the analysis of [19] to allow for both open and closed string fluctuations. For some applications of warped compactifications such as D-brane inflation where both open and closed string degrees of freedom are simultaneously dynamical, this extension is in fact inevitable.

The aim of this note is to take a first step in this direction by considering a D3-brane in a strongly warped background. We include the fluctuations of both the D3 moduli and the closed string moduli in our analysis and derive the combined Kähler potential. Among the subtleties in deriving the warped effective action is a correct identification of the gauge invariant perturbations [1, 16]. A naive dimensional reduction without dividing the field space metric by gauge redundancies leads to a conjectured expression [26] that does not minimize the inner product of metric perturbations over each gauge orbit. We identified the

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[1] See [6, 7, 8, 9] for some early attempts of realizing [5], albeit with supersymmetry, in string theory.

[2] The inflaton potential and the flavor problem in gravity mediation are two examples in which the precise form of the Kähler potential plays an important role.

gauge invariant perturbations for the combined open and closed modul space and obtained the resulting Kähler potential. Although our main focus is the combined open and closed string system, our results have clarified and elucidated some previous closed string computations. In particular, we found a gauge that simplifies not only the worldvolume action of the D3-branes at strong warping, but also in computing the Kähler metric for the closed string fluctuations.

This paper is organized as follows. In Section 2, we review and clarify the Hamiltonian formalism which we used to extract the kinetic terms of moduli fields in warped compactification. In particular, we discuss how one can carry out the computation in different gauges, emphasizing that the Kähler metrics are gauge invariant while the Hamiltonian constraints (except for special cases discussed below) depend on the gauge choice. We also suggest a convenient gauge useful for deriving the kinetic terms of light scalar fields that do not develop a classical potential[3]. In Section 3, we extend our approach to include both fluctuating closed string and D3-brane moduli. The convenient gauge choice introduced in Section 2 also turns out to diagonalize the kinetic terms for the open plus closed string system. As an illustration, we present the combined Kähler potential involving the D3 moduli and the universal Kähler modulus. We also comment on how the rho problem can be solved even in the strong warping limit when backreaction of the D3-branes is taken into account. We end with some discussions in Section 4.

2 Warped Compactification and Hamiltonian Formalism

We first review the relevant background about extracting the kinetic terms of moduli fields in warped compactification, following [1, 14, 16]. Our starting point is the supergravity action for the type IIB string theory in Einstein frame. We follow the convention in [9]:

$$
S_{IIB}^{\mathrm{E}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g_{10}} \left\{ \mathcal{R}^{(10)} - \frac{\partial_M \tau \partial^M \bar\tau}{2(\mathrm{Im}\tau)^2} - \frac{G_3 \cdot \bar G_3}{12\mathrm{Im}\tau} - \frac{\tilde F_5^2}{4\cdot 5!} \right\} + \frac{1}{8i\kappa_{10}^2} \int \frac{C_4 \wedge G_3 \wedge \bar G_3}{\mathrm{Im}\tau} + S_{\mathrm{loc.}} \, .
$$
(1)

Here τ is the complex dilaton-axion, $G_3 = F_3 - \tau H_3$ is the complex three form flux, C_4 is the RR four form and $\tilde F_5 = dC_4 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3$ is the self-dual five form flux, satisfying $\tilde F_5 = \star_{10} \tilde F_5$. The term $S_{\mathrm{loc.}}$ is the action of localized objects such as D-branes, whose kinematics in a warped background will be studied momentarily.

We will focus on the ten dimensional warped metric which preserves the maximal four dimensional isometries. It takes the form:

$$
ds^2 = e^{2A(y,u)} \tilde g_{\mu\nu}(x) dx^\mu dx^\nu + e^{-2A(y,u)} \tilde g_{ij}(y,u) dy^i dy^j \, .
$$
(2)

Here $e^{-4A(y,u)}$ is the warped factor and its precise form depends on the fluxes and localized sources. For definiteness, we will consider the supersymmetric backgrounds generated by

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[3]This does not exclude the possibility that those moduli are lifted by non-perturbative effects, e.g., the universal Kähler modulus in [21].

the fluxes satifying [9]:

$$e^{4A_0} = \alpha(y) \quad \text{where} \quad \tilde{F}_5 = (1 + \star_{10})[d\alpha(y) \wedge d^4x] \,, \tag{3}$$

and the Imaginary Self-Dual (ISD) condition

$$\star_6 \, G_3 = iG_3 \,. \tag{4}$$

Given the background solutions, the whole set of zero modes $\{u^I\}$, such as the complex and Kähler structure moduli, parametrize the fluctuations around them. Under the dimension reduction to the four dimensional spacetime, we can extract the kinetic energy terms for u^I in the four dimensional effective field theory by promoting them into spacetime dependent fields $u^I(x)$. The metric $G_{IJ}(u)$ on the space of zero modes parameterized by $\{u^I\}$ or "the moduli space metric" in short, can subsequently be obtained from the terms quadratic in $\partial_\mu u^I(x)$:

$$\int d^4x \sqrt{-\tilde{g}_4} \tilde{g}^{\mu\nu} G_{IJ}(u) \partial_\mu u^I \partial_\nu u^J \,. \tag{5}$$

However not all the metric fluctuation $\delta_I g_{MN}(u)$ should enter the computation of (5), as further subtleties arise. It is illuminating to pause here and draw parallel with the moduli space metric of Yang-Mills instantons (for a modern introduction, see e.g. [23]). Suppose $A_\mu^{(0)}$ is an instanton solution and consider a small fluctuation $\delta_I A_\mu$ around $A_\mu^{(0)}$, as parameterized by some moduli/collective coordinates $\{X^I\}$. For $\delta_I A_\mu$ to be a *physical* zero mode that enters the moduli space metric of instanton, it needs to satisfy both the linearized equation of motion and the orthogonality condition to other gauge transformations.

The situation with the metric zero modes is analogous. First, to ensure that the perturbed metric remains a solution to the ten-dimensional equation of motion, it is necessary for the metric fluctuations to satisfy the linearized equation of motion. Generically this fails by naively setting $\{u^I\}$ to be dynamical in (2), and more elaborated ansatz is required. The presence of fluxes and localized sources typically makes explicitly solving the linearized equation a difficult task.

Moreover for a given metric ansatz satisfying the linerized equations of motion, an infinite family of equivalent u^I-dependent metric zero modes can be futher generarated by the ten dimensional diffeomorphism transformation. This is analogous to the gauge transformation in Yang-Mills theory. As for the instantons, on the space of possible metric zero modes, the *physical* or *inequivalent* metric fluctuations which appear in the moduli space metric are the ones orthogonal to the gauge/diffeomorphism transformations. Additional constraint on $\delta_I g_{MN}$ is therefore required to ensure such an orthogonality condition.

Given the linearized equations of motion and the constraint from orthogonality, the analysis for metric zero modes hence the moduli space metric becomes seemingly involved. As it turned out, however, for extracting the modular kinetic energy terms, an elegant and efficient approach based on the familiar ADM/Hamiltonian formulation of general relativity [24] (see also [25]) was introduced in [1]. In this approach, by using a particular type of ansatz for the metric fluctuations, the linearized equations of motion containing single time derivative naturally arise as the gauge orthogonality conditions.

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The starting point for applying the Hamiltonian approach with multiple moduli $\{u^I\}$ is the following general ansatz [14]:

$$
\begin{aligned}
ds^2 \;=\;& e^{2A(y,u)+2\Omega(u)}\tilde{g}_{\mu\nu}(x)dx^\mu dx^\nu + e^{-2A(y,u)}\tilde{g}_{ij}(y,u)dy^i dy^j \\
+\;& 2e^{2A(y,u)+2\Omega(u)}(\partial_\mu\partial_\nu u^I(x)\mathbf{K}_I(y)dx^\mu dx^\nu + \partial_\mu u^I(x)\mathbf{B}_{iI}(y)dx^\mu dy^i)\,.
\end{aligned}
\tag{6}
$$

In the first line of (6), $u^I(x)$ are now dynamical; however for the spacetime dependent metric fluctuations, the terms in the second line, which are proportional to $\partial_\mu u^I(x)$ and $\partial_\mu\partial_\nu u^I(x)$ are also allowed. These additional terms are refered in the literature as the *compensators* [22, 14], and they vanish when the moduli u^I become static. Their presence is needed to ensure that the perturbed metric (6) satisfies the linearized equations of motion and they play pivotal roles in our subsequent discussions. Notice that in (6), the moduli are assumed to vary slowly, and, therefore, we only keep the linear order in both velocity $\partial_\mu u^I(x)$ and the acceleration $\partial_\mu\partial_\nu u^I(x)$. We will also take the magnitude of $u^I(x)$ to be small, as when u^Is become constant, they should correspond to small deformation of the background metric.

Under the dimensional reduction, in order to bring us back into Einstein frame where the four dimensional Planck mass is constant, here we have also included the Weyl factor $e^{2\Omega(u)}$ (see also [18]):

$$
e^{-2\Omega(u)} = \frac{\int d^6y\sqrt{\tilde{g}_6}e^{-4(y,u)}}{\int d^6y\sqrt{\tilde{g}_6}} = \frac{V_W(u)}{V_{CY}}\,.
\tag{7}
$$

It will play an important role in our later discussion about the Kähler potential of various moduli. Strictly speaking, such a definition is only valid if the unwarped metric $\tilde{g}_{ij}$ is independent of the moduli u^I; otherwise the four dimensional Planck mass would be moduli dependent. For $\tilde{g}_{ij}$ with non-trivial moduli dependence, one should therefore replace $V_{CY} = \int d^6y\sqrt{\tilde{g}_6}$ by other moduli independent fiducial volume measure, such as $(\alpha')^3$. Notice that in constrast to earlier literatures (see for example [27, 29]), where the Weyl factor (or breathing modes) and the warp factor were treated as independent quantities in the weak warping limit, here with significant warping, they are in fact related by definition.

As the metric fluctuations become time-dependent, a more illuminating way to understand the role of compensators is to view the metric (6) from the familiar ADM or Hamiltonian formulation of general relativity [1] (For a detailed review, see [25]). Applying this formalism to a ten dimensional spacetime $\mathcal{M}$ with metric g_{MN} amounts to choosing a time coordinate t and dual time-flow vector t^M, where one can decompose t^M into its normal n^M and tangential η^M components with respect to a nine dimensional space-like hypersurface of constant time Σ_t. We can follow the standard procedures to define the *lapse function N*:

$$
N = -g_{MN}t^M n^N\,,
\tag{8}
$$

and the *shift vector*:

$$
\eta^M = h^{MN}\eta_N = t^M - Nn^M\,,
\tag{9}
$$

Here h_{MN} is the pullback metric of g_{MN} onto Σ_t, and they are related by $h_{MN} = g_{MN}+n_M n_N$, we can therefore use (h_{MN}, N, η^M) to recast the ten-dimensional metric into

$$
ds^2 = -N^2 dt^2 + h_{MN}\left(dx^M + \eta^M dt\right)\left(dx^N + \eta^N dt\right)\,.
\tag{10}
$$

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Comparing (10) and (6), we clearly see the analogy. Up to the linear order in $\partial_\mu u^I$ and $\partial_\mu \partial_\nu u^I$, the compensator can be identified with the shift vector (9). More concretely, consider $x^0 = t$ with $g_{tt} = -g_{00} > 0$ in (6), the lapse function is given as $N = (g_{tt})^{1/2}$. The shift vectors and compensators are simply identified as $\eta_M = g_{0M}$, $M = 1, \ldots, 9$, or in terms of components[4]:

$$\eta_\alpha = e^{2A(y,u)+2\Omega(u)} \partial_\alpha \partial_0 u^I \mathbf{K}_I(y)\,, \quad \eta_i = e^{2A(y,u)+2\Omega(u)} \partial_0 u^I \mathbf{B}_{iI}(y)\,, \quad \alpha = 1,2,3\,, \ i = 4\ldots,9. \tag{11}$$

It is now also immediately clear that the physical metric fluctuation should be normal to the space-like surface Σ_t, and it is given by the extrinsic curvature K_{MN}:

$$K_{MN} = \frac{1}{2}\mathcal{L}_n h_{MN} = \frac{1}{2}(g^{tt})^{1/2}\left(\frac{dh_{MN}}{dt} - D_M\eta_N - D_N\eta_M\right). \tag{12}$$

Here the covariant derivative D_M is defined with respect to the nine-dimensional metric h_{MN}, which is identified with $h_{MN} \equiv g_{MN}$, $M, N \neq 0$. For the remainder of the paper therefore, $M, N = 1, \ldots, 9$ unless otherwise stated. Notice the similarity between (12) and the physical zero mode for the Yang-Mills instanton, η^M essentially plays the role of the parameter in the compensating gauge transformation. One can also interpret it as combining the naive variation of the metric h_{MN} with respect to t and the diffeomorphism transformation on Σ_t.

Using (12), one can express the Lagrangian density/Ricci scalar in terms of K^{MN} given by

$$\mathcal{L}^G_{\text{kin}} = \sqrt{-g_{10}}\left(R^{(9)} + K^{MN}K_{MN} - (h^{MN}K_{MN})^2\right). \tag{13}$$

The Hamiltonian approach involves defining the canonical momentum π^{MN} conjugate to h_{MN}:

$$\pi^{MN} = \frac{\partial\mathcal{L}_G}{\partial\dot{h}_{MN}} = \sqrt{h}\left(K^{MN} - h^{MN}(h^{PQ}K_{PQ})\right). \tag{14}$$

The Hamiltonian is then obtained from the definition $\mathcal{H}^G = \pi^{MN}\dot{h}_{MN} - \mathcal{L}^G_{\text{kin}}$, written in terms of π^{MN}:

$$\mathcal{H}^G = \sqrt{-g_{10}}\left(-R^9 + h^{-1}\pi^{MN}\pi_{MN} - \frac{1}{8}h^{-1}(\pi_{MN}h^{MN})^2\right) + 2\pi^{MN}D_M\eta_N. \tag{15}$$

It can be shown that in the Hamiltonian, up to a total derivative term, η^N only appears as a non-dynamical Lagrange multiplier imposing the following constraint:

$$D^M(h^{-1/2}\pi_{MN}) = 0. \tag{16}$$

We will explain the significance of (16) momentarily. The kinetic terms from the remaining Hamiltonian (modulo the nine-dimensional Ricci scalar term $R^{(9)}$) with (16) imposed are then given by [1]:

$$\mathcal{H}^G_{\text{kin}} = \int d^9x\sqrt{-g_{10}}h^{-1}\left(\pi^{MN}\pi_{MN} - \frac{1}{8}\pi^2\right) = \int d^9x\sqrt{h}(g_{tt})^{1/2}h^{-1/2}\pi_{MN}K^{MN}. \tag{17}$$

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[4]Here we use $\alpha, \beta = 1, 2, 3$ to denote the three spatial directions in order to distinguish $\mu, \nu = 0, 1, 2, 3$, which include time-like direction.

In the second equality, we highlighted that $\mathcal{H}_{\text{kin}}^G$ is the gravitational analog of the $p\dot{q}$ term in the classical dynamics.

To understand the significance of the constraint (16), we recall that there is arbitrariness in our choice of Σ_t, hence the induced metric h_{MN}. If ψ represents the diffeomorphism of Σ_t, h_{MN} and $\psi^* h_{MN}$ should represent the same physical configuration. In other words, our physical configuration is a space of equivalence classes of metrics h_{MN} on Σ_t, where the equivalence is given by the diffeomorphism ψ. This is precisely what is referred in the literature as the "superspace". Consider a nine dimensional diffeomorphism transformation of Σ_t acting on K^{MN}:

$$K^{MN} \longrightarrow K^{MN} - \frac{1}{2}(g^{tt})^{1/2}(D^M V^N + D^N V^M), \tag{18}$$

The kinetic term $\mathcal{H}_{\text{kin}}^G$ is therefore shifted by a term:

$$- \int d^9 x \sqrt{h} D^M V^N (\pi_{MN} h^{-1/2}) = \int d^9 x \sqrt{h} \left(V^N D^M (h^{-1/2} \pi_{MN}) - D^M (V^N h^{-1/2} \pi_{MN}) \right). \tag{19}$$

As we choose the superspace as physical configuration space, the conjugate momentum π_{MN} must ensure that (19) vanishes. Since the second term in (19) is merely a total derivative and V_M is an arbitrary vector parameterizing the diffeomorphism transformation, the constraint (16) is precisely the condition needed to ensure (19) vanishes. In other words, with respect to the inner product defined in (17), (16) is the condition to guarantee the metric fluctuation to be *physical*, i.e. orthogonal to arbitrary diffeomorphism transformation.

One can also verify that, by substituting the given ansatz (6), the orthogonality condition (16) is equivalent to the ten dimensional linearized Einstein equations, since they simply come from varying the metric component g_{0M}. Explicitly, we can make the following identifications:

$$(g^{tt})^{1/2} D_N (h^{-1/2} \pi^{NM}) = \delta R^{0M} = \delta G^{0M} = 0, \quad M = 1, \ldots, 9. \tag{20}$$

In the second equality we have used the fact that in the limit of static u^I, $g_{0M} = g_{M0} = 0$ and we only keep quadratic flucutuations in the action. In [16], it was also pointed out that as δG_{0M} Einstein equations only contain single time derivative, they act as the constraints which should be satisfied by any consistent solutions at all time. In the Hamiltonian formalism, such constraints are made manifest and their role is elegantly explained. Calculationally, the identifications (20) allows us to recycle some of the calculations for the linearized Ricci tensors [14], when expressing the constraint equation (16) explicitly. This also tells us that in the presence of additional localized energy sources such as D-brane, the linearized Einstein hence the constraint equations get modified.

To apply the Hamiltonian formalism reviewed earlier to extract the moduli space metric, we consider the simpler case that $\partial_\mu u^I \equiv \delta_\mu^0 \dot{u}^I(t)$. The ten dimensional kinetic term is then given by [5]

$$S_{\text{kin}}^G = \frac{1}{2\kappa_{10}^2} \int dt \mathcal{H}_{\text{kin}} = \frac{1}{8\kappa_{10}^2} \int dt \dot{u}^I \dot{u}^J \int d^9 x \sqrt{-g_{10}} g^{tt} \delta_I h^{MN} \delta_J \pi_{MN}. \tag{21}$$

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[5]To be more precise, in our analysis for the kinetic term, we are only keeping the quadratic fluctuations. At this order the integration measure $\sqrt{-g_{10}}$ or $\sqrt{h}$ multiplying the $\pi_{MN} \pi^{MN}$ and π^2 terms do not carry

Here the terms $\delta_I h_{MN}$ and $\delta_I \pi_{MN}$ are related to K_{MN} and π_{MN} by:

$$K_{MN} = \frac{1}{2}(g^{tt})^{1/2}\dot{u}^I \delta_I h_{MN}\,, \quad \pi_{MN} = \frac{1}{2}\sqrt{h}(g^{tt})^{1/2}\dot{u}^I \delta_I \pi_{MN}\,, \tag{22}$$

where we have factored out the moduli dependence such that $dh_{MN}/dt = \dot{u}^I(\partial h_{MN}/\partial u^I)$ and $\eta_M = \dot{u}^I \eta_{IM}$. The moduli space metric, denoted $G_{IJ}(u)$ is then given by:

$$G_{IJ}(u) = \frac{1}{8\kappa_{10}^2}\int d^6y\sqrt{\tilde{g}_6}e^{-4A+2\Omega}\delta_I \pi_{MN}\delta_J h^{MN}\,, \tag{23}$$

such that the kinetic term can be rewritten as:

$$S_{\text{kin}}^G = \int d^4x\sqrt{-\tilde{g}_4}\tilde{g}^{tt}\dot{u}^I\dot{u}^J G_{IJ}(u). \tag{24}$$

The constraint equation (16) can also be written in terms of $\delta_I \pi_{MN}$:

$$D^M((g^{tt})^{1/2}\delta_I \pi_{MN}) = 0\,. \tag{25}$$

As an illustration to the earlier general discussion on the Hamiltonian formalism, let us pause here to discuss the explicit metric gauge choices where the computations will be performed. The presence of the compensators $\mathbf{K}_I(y)$ and $\mathbf{B}_{iI}(y)$ are associated with the spacetime dependent metric fluctuations. As pointed out in [14], we can use the freedom to parametrize the flucutuations to eliminate them by performing a ten dimensional diffeomorphism transformation with the sacrifice that the internal metric and warp factor should be simultaneously changed. As an example, we can consider for $u^I(t)$, the following diffeomorphism generated by ζ_A on the metric (6):

$$\mathrm{K} - \text{gauge}: \quad \zeta_\mu = 0\,, \quad \zeta_i = -e^{2A(y,u)+2\Omega(u)}\mathbf{B}_{iI}(y)u^I(t)\,. \tag{26}$$

Here the transformation is given by the covariant derivative defined from the full ten-dimensional metric (6). The metric (6) is then transformed into:

$$\begin{aligned}
ds_{(K)}^2 &= e^{2A_K(y,u)+2\Omega_K(u)}\tilde{g}_{\mu\nu}(x)dx^\mu dx^\nu + e^{-2A_K(y,u)}\tilde{g}_{ij}^{(K)}(y,u)dy^i dy^j \\
&+ 2e^{2A_K(y,u)+2\Omega_K(u)}(\ddot{u}^I\mathbf{K}_I(y)(dt)^2)\,.
\end{aligned} \tag{27}$$

$$e^{\pm 2A_K(y,u)} = e^{\pm 2A(y,u)}(1 \pm 2\zeta_i\partial^i A)\,,$$

$$\tilde{g}_{ij}^{(K)}(y,u) = \tilde{g}_{ij}(y,u) - \tilde{\nabla}_i(e^{2A(y,u)}\zeta_j) - \tilde{\nabla}_j(e^{2A(y,u)}\zeta_i)\,. \tag{28}$$

where $\tilde{\nabla}_i$ is the covariant derivative with respect to the unwarped six dimensional metric $\tilde{g}_{ij}(y,u)$. Notice that the transformation (26) shuffles the $\mathbf{B}_{iI}(y)$ dependent off-diagonal terms into both the internal metric and the warp factor. In such a gauge where $\tilde{g}_{ij}^{(K)}(y,u)$ contains explicit moduli-depedence, one should replace the unwarped volume V_{CY} in (7)

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any explicit u dependence. However if there are additional u-dependence in the overall action, arising from the nine-dimensional curvature $R^{(9)}$ or flux induced potential, the $\partial_\mu\partial_\nu u^I$ terms in $\sqrt{-g_{10}}$ can in principle also give extra contribution to the kinetic energy via integration by parts.

by a moduli-independent fiducial volume to define the Weyl factor $e^{-2\Omega_K}$, and define the warped volume with respect to e^{-4A_K} and $\tilde{g}_{ij}^{(K)}$. For later convenience, we will refer to the metric ansatz (27) as the *"K-gauge"*. Note that such a gauge choice is still not unique since one could perform the gauge transformation $\zeta_i = ue^{2A+2\Omega}\partial_i\Lambda$, $\zeta_\mu = -\partial_\mu ue^{2A+2\Omega}\Lambda$ to further shuffle the compensator term $\mathbf{K}$, into warp factor e^{4A} as well as the internal metric $\tilde{g}_{ij}$. In the later application, we use this residual gauge freedom to define the universal Kähler moduli in the conventional way.

In contrast, the gauge transformation $\zeta_i = 0$, $\zeta_\mu = \partial_\mu ue^{2A+2\Omega}\Lambda$ does not generate changes in the internal metric as well as the warp factor while it shuffles $\mathbf{B}_i$ and $\mathbf{K}$ [14, 18]. This always enables us to go from K-gauge to the gauge where $\mathbf{K} \to 0$, $\mathbf{B}_i \to \partial_i\mathbf{K}$ without changing $\tilde{g}_{ij}$ nor e^{4A}. This is consistent with the fact that the linearized equations of motion [14] (or constraint equations in the Hamiltonian formulation) always accompany the combination $\mathbf{B}_i - \partial_i\mathbf{K}$. Note, however, one could not conversely gauge away arbitrary $\mathbf{B}_i$ in this way because $\mathbf{B}_i$ should be a total derivative to do so.

From the perspective of the Hamiltonian formulation, one can calculate the extrinsic curvature K_{MN} (12) in the general case (6) and in the K-gauge (27), and demonstrate that the ten dimensional diffeomorphism transformations relating them can be decomposed into a nine dimensional diffeomorphism transformation acting on $\delta_I h_{MN}$ and a reparametrization of g^{tt}. Using the constraint (25), one can show that, with the time reparametrization in $\int dt\sqrt{\tilde{g}_{tt}}\tilde{g}^{tt}\dot{u}^I\dot{u}^J$ of the kinetic term S_{kin}^G (24) which can be ensured when one imposes time reparametrization invariance, the moduli space metric $G_{IJ}(u)$ is indeed invariant under different gauge choices. In practical terms, if one has a metric ansatz which consistently solves the constraint equations (16) and other linearized equations of motion, then all other metric ansatz relating to it via ten dimensional diffeomorphism transformation would yield identical moduli space metric under the dimension reduction. However to identify the precise metric ansatz corresponding to the fluctuation of a given modulus, additional information such as the preservation of certain global symmetries, is generally required.

3   Coupling to D3 branes

In this section we would like to consider coupling a D3 brane to the warped closed string background given by metric (6). Our aim here is to derive the Kähler potential for the position modulus of a mobile D3 brane. In particular we will consider the simplified situation where only the unversal Kähler modulus $c(x)$ (and its imaginary partner) and the D3 brane position moduli $Y^i(x)$ are fluctuating. This is justified since a D3 brane does not source other moduli such as the dilaton τ or C_2. Earlier, the Hamiltonian provides a natural inner product on the space of metric fluctuations, where the orthogonality constraint to the unphysical diffeomorphism transformation can be imposed by the compensators. The question here would therefore be: What is the corresponding inner product on the vector space spanned by both closed and open string fluctuations, and the associated orthogonality constraints?

Let us clarify the steps being taken here. We start with the dynamical warped background given by (6), and introduce a single spacetime filling D3 brane at a point Y on the compact six manifold. The bosonic fluctuations of the D3 brane in such a background are encoded in

the DBI+CS action $S_{\rm D3}$

$$S_{\rm D3} = S_{\rm DBI} + S_{\rm CS} = -T_3 \int_{\mathcal{W}_4} d^4\xi \sqrt{-\det({\rm P}[g])} + T_3 \int_{\mathcal{W}_4} {\rm P}\left[C_{(4)}\right] . \tag{29}$$

Here ${\rm P}[g]$ and ${\rm P}\left[C_{(4)}\right]$ are the pullbacks of the bulk metric g_{MN} and RR four form $C_{(4)}$ into the D3 brane world volume $\mathcal{W}_4$. Here for our purpose of deriving the Kähler potential, we will ignore the worldvolume gauge fields and the pullback of the NS-NS two form, as their effects will mainly be modifying the partial derivatives into covariant ones [30]. To incorporate (29) into the full ten dimensional action (1), the localized action is then given by

$$S_{\rm loc.} = 2\kappa_{10}^2 \int d^6y \delta^{(6)}(y - Y)S_{\rm D3}[Y] . \tag{30}$$

Here we will present a complete derivation for the universal Kähler potential, and allow for both open and closed string degrees of freedom to fluctuate in the full action (1). We will extract the relevant pieces in the D3 action (29), and then combine with the closed string action to obtain the full kinetic terms. In particular, due to the presence of the open string fluctuations, the orthogonality constraint (16) will be modified. Although we will later focus on the case of the universal Kähler modulus, parts of our discussion will be applicable for other closed string moduli coupling to the open string degrees of freedom.

Let us focus on the kinetic terms of the scalar fields in the DBI Lagrangian (29), and the relevant terms are given by the pull-back of the metric (6)

$$\begin{aligned}
{\rm P}(g)_{\mu\nu} &= e^{2A(Y,u)+2\Omega(u)}\left\{\tilde{g}_{\mu\nu}(x) + 2\partial_\mu\partial_\nu u^I(x)\mathbf{K}_I(Y) + 2\mathbf{B}_{iI}(Y)\partial_\mu u^I(x)\partial_\nu Y^i\right\} \\
&+ e^{-2A(Y,u)}\tilde{g}_{ij}(Y,u)\partial_\mu Y^i\partial_\nu Y^j .
\end{aligned} \tag{31}$$

Here the indices μ, ν run over the D3 brane worldvolume coordinates and we have made the static gauge choice $\xi^\mu = x^\mu$. Notice that the warped factor $e^{-4A(Y,u)}$ and the fiducial metric $\tilde{g}_{ij}(Y,u)$ are evaluated at the locus of the D3 brane in the compact Calabi-Yau manifold.

The determinant of (31) can be readily evaluated by using

$$\sqrt{\det(1 + M)} = 1 + \frac{1}{2}{\rm Tr}(M) - \frac{1}{4}{\rm Tr}(M^2) + \frac{1}{8}({\rm Tr}(M))^2 + \ldots . \tag{32}$$

which yields

$$-\frac{\mathcal{L}_{\rm kin}^{\rm DBI}}{T_3\sqrt{-\tilde{g}_4}} = \frac{e^{2\Omega(u)}}{2}\tilde{g}_{ij}(Y,u)\partial_\mu Y^i\partial^{\tilde{\mu}} Y^j + e^{4(A(Y,u)+\Omega(u))}(\mathbf{B}_{iI}(Y)\partial_\mu u^I(x)\partial^{\tilde{\mu}} Y^i) + \ldots \tag{33}$$

Here we have eliminated the vaccum energy $\propto e^{4(A(Y,u)+\Omega(u))}$ in (33), as it will be cancelled by the pullback of C_4 in the CS term in a supersymmetric background specified by (3) and (4) [9], and we have only kept terms up to quadratic order in the spacetime derivatives[6]. The indice $\tilde{\mu}$ here is to highlight that the summation in done using the unwarped four dimensional

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[6]To be clear, in factoring out the four dimensional spacetime metric from (31), we have included $\partial_\mu\partial_\nu u^I\mathbf{K}_I(y)$ term. Since we are only keeping quadratic fluctuations in the action, for most places, the contribution from $\partial_\mu\partial_\nu u^I\mathbf{K}_I(y)$ is neglected. We come back to this point in section 4.

background metric $\tilde{g}_{\mu\nu}$. The first term is the usual kinetic term for the transverse fluctuations $Y^i(x)$, whereas the others represent the non-trivial coupling between the closed string and the open string fluctuations. To complete the list, one would also need to include the fluctuations in the Chern-Simons term S_{CS}, and couple them to the relevant RR kinetic terms in the supergavity action. Instead of getting into that, however, one can already consider the possible modification to the orthogonality condition imposed by the metric compensators. This can be analysed from the Hamiltonian approach reviewed earlier, or from the more conventional Lagrangian approach used in [16]. The equivalence between the two approaches relies on the fact that, in the presence of a D3 brane, for the metric ansatz of the form, the orthogonality conditions with the appropriately defined inner product remains the same as the linearized equations of motion.

To generalize the Hamiltonian approach to include D3 brane, we will again focus on the Einstein term in the SUGRA action and the kinetic terms given by the expansion of the DBI action (33). Furthermore, we will assume for the moment, the RR fluctuations are frozen out. The natural inner product can then be constructed from considering the following integral

$$\mathbf{H}^{\text{All}} = \int d^9x \left\{ \dot{h}_{MN} \pi^{MN} - \mathcal{L}_{\text{kin}}^G + 2\kappa_{10}^2 \delta^{(6)}(y-Y)\mathcal{H}^{\text{DBI}} \right\}, \tag{34}$$

which is a natural generalization of the closed string Hamiltonian (17). Here the canonical momentum π^{MN} is as defined in (14), whereas $\mathcal{L}_{\text{kin}}^G$ and $\mathcal{L}_{\text{kin}}^{\text{DBI}}$ are given by (13) and (33). We have also restored the six dimensional delta function $\delta^{(6)}(y-Y)$ and defined the Hamiltonian for the D3 brane $\mathcal{H}^{DBI}$:

$$\begin{aligned}
\mathcal{H}^{DBI} &= g_{ij}(P^j \dot{Y}^i) - \mathcal{L}_{\text{kin}}^{\text{DBI}} \\
&= -\eta_i P^i + \frac{1}{2}\frac{g_{ij}g_{tt}}{T_3\sqrt{-g_4}}P^i P^j + (\partial_\alpha Y^i \text{ dependent terms}),
\end{aligned} \tag{35}$$

where $\partial_\alpha Y^i$ denotes the partial derivative with respect to the three external spatial coordinates[7]. The canonical momentum P^i is given by:

$$P_i = \frac{\partial \mathcal{L}_{\text{kin}}^{\text{DBI}}}{\partial \dot{Y}^i} = T_3\sqrt{-g_4}(g^{tt}(\dot{Y}_i + \eta_i)) = T_3\sqrt{-g_4}g^{tt}K_i, \tag{36}$$

note that we have taken the convention $g_{tt} = -g_{00} > 0$. Similar to the extrinsic curvature K^{MN} which is the physical metric fluctuations orthogonal to the spacelike hypersurface Σ_t, the vector K^i has the natural geometric interpretation as the vector fluctuation orthogonal to Σ_t, and transforms under the nine-dimensional diffeomorphism of Σ_t. In (34), the compensators η_α and η_i defined in (11) again appear as non-dynamical Lagrangian multipliers. Due to the presence of D3 transverse fluctuations, however, the constraints imposed by η_α and η_i now become:

$$D^M \left(h^{-1/2}\pi_{M\alpha} \right) = 0, \tag{37}$$

$$D^M(h^{-1/2}\pi_{Mi}) + \kappa_{10}^2\delta^{(6)}(y-Y)\frac{P_i}{\sqrt{h}} = 0. \tag{38}$$

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[7]Notice here that in the derivation of the Hamiltonian (35), we have neglected the terms proportional to $(\partial_\mu u^I B_{iI}(y))^2$, in the same approximation made in the Hamiltonian formulation of the closed string moduli.

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Notice that we have assumed that $\tilde{g}_{\alpha 0} = \tilde{g}_{0\alpha} = 0$, as the external spacetime is assumed to be maximally symmetric, so the external constraint (37) does not receive δ-function corrections[8]. One can also verify that (37) and (38) are equivalent to the linearized equations of motion in the presences of D3 branes:

$$\delta G_{0M} = \kappa_{10}^2 \delta T_{0M}^{(D3)}, \quad M = 1, \ldots, 9. \tag{39}$$

To see that $\mathbf{H}^{\mathrm{All}}$ (34) with the constraint (38) imposed is indeed the correct inner product, let us first write out the remainder of (34) after integrating out η_N:

$$\mathbf{H}^{\mathrm{All}} = \int d^9 x \sqrt{h} \left[(g_{tt})^{1/2} h^{-1/2} \pi_{MN} K^{MN} + \kappa_{10}^2 \frac{\delta^{(6)}(y - Y)}{\sqrt{h}} K^i P_i \right]. \tag{40}$$

Here we have again taken the simpler case where $\partial_\mu u^I(x) = \delta_\mu^0 \dot{u}^I$ and $\partial_\mu Y^i = \delta_\mu^0 \dot{Y}^i$ for extracting the kinetic energy. Now we can consider a nine-dimensional diffeomorphism transformation of Σ_t acting on K^{MN} and K^i, as generated by vector V^N, such that K^{MN} transforms as in (18) whereas:

$$K^i \longrightarrow K^i + V^i, \tag{41}$$

Substituting (18) and (41) into (40), one can show that, up to a total derivative term, the condition for the inner product to vanish on the unphysical diffeomorphism transformation is given by:

$$\int d^9 x \sqrt{h} \left\{ V^\alpha D^M \left(h^{-1/2} \pi_{M\alpha} \right) + V^i \left[D^M \left(h^{-1/2} \pi_{Mi} \right) + \kappa_{10}^2 \delta^{(6)}(y - Y) \frac{P_i}{\sqrt{h}} \right] \right\} \tag{42}$$

Since V^N is an arbitrary vector, the constraint (38) in the compact directions combines with the one in the spacetime directions (37) ensure that the orthogonality conditions under diffeomorphism transformation are indeed imposed. Once again, due to such invariance of $\mathbf{H}^{\mathrm{All}}$ (40), we can evaluate the moduli space metric in the convenient gauge, up to a reparametrization of the time variable.

Having discussed how to couple a D3 brane to the universal Kähler modulus in the context of the Hamiltonian formalism, for the explicit computations of the moduli space metric, we will utilitize the diffeomorphism transformation on the metric (18) and the vector (41) to work in the K-gauge given in (27). This gauge holds distinct advantage that there are no cross couplings between terms such as $\partial_\mu u^I$ and $\partial_\mu Y^i$ as we can see in (33). Here in our analysis we will mostly work in the probe limit and neglect the change to the background metric caused by the motion of the mobile D3, and hence Y^i is not a fluctuating modulus of the metric. We can then read off the kinetic energies for the universal Kähler modulus c from the 10D Einstein term and for the D3 brane modulus from the DBI action. We will also comment on how backreaction of the D3 brane can be incorporated in Hamiltonian formalism at the end of the section.

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[8]In fact, to ensure that this equation is satisfied globally over a compact space, one also needs to include as usual a background charge density on the RHS of (38).

Explicitly the metric fluctuations $\delta_I h_{MN}$ in the K-gauge (27) take the following form:

$$\delta_I h_{\alpha\beta}(x,u) = 2\left(\frac{\partial A(y,u)}{\partial u^I} + \frac{\partial \Omega(u)}{\partial u^I}\right) e^{2A(y,u)+2\Omega(u)}\tilde{h}_{\alpha\beta}(x)\,, \tag{43}$$

$$\delta_I h_{ij}(y,u) = -e^{-2A(y,u)}\left(2\frac{\partial A(y,u)}{\partial u^I}\tilde{g}_{ij}(y,u) - \frac{\partial \tilde{g}_{ij}(y,u)}{\partial u^I}\right)\,. \tag{44}$$

where $\tilde{h}_{\alpha\beta}(x) = \tilde{g}_{\mu\nu}(x)$ for $\mu,\nu = 1,2,3$. Here for the simplicity of notation, we have omitted the subscript "K" in the various quantities in (27). The canonical momenta $\delta_I\pi_{MN}$ are then given by:

$$\delta_I\pi_{\alpha\beta} = \left(8\frac{\partial A(y,u)}{\partial u^I} - 4\frac{\partial \Omega(u)}{\partial u^I} - \tilde{g}^{kl}(y,u)\frac{\partial \tilde{g}_{kl}(y,u)}{\partial u^I}\right)h_{\alpha\beta}(x,u)\,, \tag{45}$$

$$\delta_I\pi_{ij} = \left(4\frac{\partial A(y,u)}{\partial u^I} - 6\frac{\partial \Omega(u)}{\partial u^I} - \tilde{g}^{kl}(y,u)\frac{\partial \tilde{g}_{kl}(y,u)}{\partial u^I}\right)g_{ij}(y,u) + e^{-2A(y,u)}\frac{\partial \tilde{g}_{ij}(y,u)}{\partial u^I} \tag{46}$$

$$P_i = T_3\sqrt{-g_4}g^{tt}\dot{Y}_i\,. \tag{47}$$

Let us consider the constraint equations (37) and (38), in the probe limit we will neglect the perturbation due to D3 on the warped metric, hence the $\delta^{(6)}(y-Y)$ term in (38). One can next show that the constraint (37) can be trivially satisfied, while (38) yields:

$$D^M((g^{tt})^{1/2}\delta_I\pi_{Mi})$$
$$= -(g^{tt})^{1/2}\left(e^{4A}\partial_i\partial_I e^{-4A(y,u)} + \tilde{\nabla}^j(\partial_I\tilde{g}_{ij} - \tilde{g}_{ij}(\tilde{g}^{kl}\partial_I\tilde{g}_{kl})) + 4\partial^{\tilde{j}}A\partial_I\tilde{g}_{ij} - 2\partial_i A\tilde{g}^{kl}\partial_I\tilde{g}_{kl}\right) = 0\,. \tag{48}$$

Now for the case of universal Kähler modulus $u^I = c$, a viable solution to (48) is then given by:

$$\partial_c\tilde{g}_{ij} = 0\,, \quad e^{-4A(y,c)} = e^{-4A_0(y)} + c\,. \tag{49}$$

Notice that one can in fact replace c by an arbitrary function $f(c)$, but this will become a mere field redefinition of the universal Kähler modulus. This solution (49) fits well with the usual physical interpretation that the universal Kähler modulus c corresponds to an overall rescaling of the internal space and preserve all of its isometries, and hence unwarped metric $\tilde{g}_{ij}$ should not have explicit dependence on c. Furthermore c remains massless even in the presence of fluxes, which can only be achieved if c (or more generally $f(c)$) appears as an undetermined integration constant in the defining equation of the warp factor.

At this point, we emphasize that the solution (49) is by no means unique. At least, we could obtain a family of solutions generated by the gauge transformation $\zeta_i = ce^{2A+2\Omega}\partial_i\Lambda$, $\zeta_\mu = -\partial_\mu ce^{2A+2\Omega}\Lambda$ that preserves the K-gauge condition. We regard $\partial_c\tilde{g}_{ij} = 0$ as a gauge condition to define our universal Kähler moduli in a conventional way. In other words, the universal Kähler moduli is defined as an overall scaling of the metric only in this specific K-gauge. With a different gauge choice, the independence of the internal metric $\tilde{g}_{ij}$ on c no longer holds in general.

It is useful to note that our background metric with no D3-brane is diffeomorphic to the one presented in [18]. To see this, we can perform the gauge transformation $\zeta_i = 0$,

$\zeta_\mu = ce^{2A+2\Omega}\partial_i\mathbf{K}$. As we discussed before, such a diffeomorphism does not change $\tilde{g}_{ij}$ and e^{4A}. As a consequence, the constraint equations derived there should be equivalent to ours with no change of variables albeit it is much more tedious to obtain them directly in their gauge.[9] Once we introduce the D3-brane, however, the gauge transformation acts on the DBI action non-trivially, and the advantage to use the K-gauge becomes distinctive.

Using (49), which implies that the internal unwarped metric is independent of the universal Kähler modulus c, we can write down an explicit expression for the Weyl factor $e^{-2\Omega}$ from the definition (7):

$$e^{-2\Omega(c)} = c + \frac{V_W^0}{V_{CY}}. \tag{50}$$

where $V_W^0 = \int d^6y\sqrt{\tilde{g}_6}e^{-4A_0(y)}$. Combining (49) and (50), we can now write down the kinetic terms for both the universal Kähler modulus c and the D3 brane modulus Y^i using (40):

$$S_{\text{kin.}}^{\text{All}} = \frac{1}{2\kappa_{10}^2}\int dt\mathbf{H}^{\text{All}} = \frac{3}{\kappa_4^2}\int d^4x\sqrt{-\tilde{g}_4}\left(\frac{1}{(2e^{-2\Omega})^2}\tilde{g}^{tt}(\dot{c})^2 + \frac{T_3\kappa_4^2}{3}\frac{1}{(2e^{-2\Omega})}\tilde{g}^{tt}\tilde{g}_{ij}\dot{Y}^i\dot{Y}^j\right), \tag{51}$$

where we have used the relation $\kappa_{10}^2 = \kappa_4^2 V_{CY}$. In contrast with [1, 18], here in deriving (51), it is crucial to keep the $\delta_c\pi_{\alpha\beta}$ contribution to kinetic energy, as we do not see obvious reasons for setting it to zero from the constraint equations. So far in the K-gauge we used (27), the explicit compensator dependence $\propto \ddot{c}\mathbf{K}_c$ only appears in g^{tt} component. Its explicit form is not determined through the constraint equations and does not enter the kinetic terms at the order of quadratic fluctuations in the absence of the potential. However even for $c \equiv c(t)$, it is crucial to have a non-vanishing $\mathbf{K}_c(y)$ to satisfy the $(1,1)$ components of the linearized Einstein equation:

$$\tilde{\nabla}^2\mathbf{K}_c(y) = e^{-4A_0(y)} - \frac{V_W^0}{V_{CY}}. \tag{52}$$

Here we have used the explicit expression for $e^{-4A(c)}$ (49) and $e^{-2\Omega(c)}$ (50) to simplify the expression. Similar equations were also noticed in [14, 18]. As will be discussed later, for moduli which develop a potential (e.g., complex structure moduli in flux compactification), a non-trivial $\mathbf{K}_c(y)$ can give rise to an additional contribution to the kinetic term for such moduli.

From (51), we can finally write down the Kähler potential:

$$\kappa_4^2\mathcal{K}(\rho, Y) = -3\log\left[\rho + \bar{\rho} - \gamma k(Y,\overline{Y}) + 2\frac{V_W^0}{V_{CY}}\right], \quad \gamma = \frac{T_3\kappa_4^2}{3}, \tag{53}$$

where V_W^0 denotes the warped volume in the absence of the D3 brane and and $k(Y,\overline{Y})$ is the little Kähler potential for the unwarped internal manifold. The holomorphic volume modulus ρ is defined to be:

$$\rho = \left(c + \frac{\gamma}{2}k(Y,\overline{Y})\right) + i\chi. \tag{54}$$

where the axionic partner χ of c comes from dimension reduction of C_4. Our definition of ρ was motivated by a non-trivial $U(1)$ fibration of χ over the Y-moduli space. This arises

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[9]We would like to thank A. Frey for helpful communication on this point.

from the pullback of C_4 onto the D3 brane world volume and manifest itself at weak warping in following transformation [28][10]:

$$\rho \to \rho + \gamma f(Y), \quad \bar{\rho} \to \bar{\rho} + \gamma \overline{f(Y)}, \quad k(Y,\overline{Y}) \to k(Y,\overline{Y}) + f(Y) + \overline{f(Y)}. \tag{55}$$

This is to ensure that the overall Kähler potential is invariant under the little Kähler transformation of $k(Y,\overline{Y})$. The Kähler potential (53) we derived at strong warping from direct dimension reduction remains consistent with such a fibration, and also allows us to determine the precise value of γ and other moduli-independent constants such as V_W^0/V_{CY}. In the absence of a D3 brane, we obtain the Kähler potential for the universal Kähler modulus which takes the same form presented in [18] though we derive it here consistently in a single gauge.

Let us finish the section on D3 brane by commenting on the issue of the "rho problem" and discussing how it can be resolved by including the backreaction of the D3-brane on the warp factor. In the probe limit we have discussed so far, the definition of the holomorphic Kähler modulus ρ yields:

$$c = \frac{1}{2}(\rho + \bar{\rho} - \gamma k(Y,\bar{Y})). \tag{56}$$

If one considers a spacetime-filling D7 brane wrapping on some supersymmetric four cycle $\mathcal{C}_4$ in the warped background (6), one can show that the gauge kinetic function is proportional to the following integral in the strongly warped limit [19]:

$$g_7^{-2} \propto \int_{\mathcal{C}_4} d^4 y \sqrt{\hat{G}_4} e^{-4A(y,c)} = \int_{\mathcal{C}_4} d^4 y \sqrt{\hat{G}_4}(c + e^{-4A_0(y)}) = cV_4 + V_4^W. \tag{57}$$

Here $\hat{G}_4$ denotes the pullback of the unwarped metric $\tilde{g}_6$ onto $\mathcal{C}_4$, and in the second equality we have used the solution (49) and V_4^W denotes the moduli independent part of the warped 4-cycle volume. Comparing (56) and (57), we see, due to the little Kähler potential $k(Y,\overline{Y})$, that g_7^2 is not the real part of a holomorphic function on the brane moduli space. Supersymmetry requires the D7 gauge kinetic function to be a holomorphic function of moduli and so as in the weakly warped situation [26, 27], there is a rho problem.

As also pointed out in [27], the resolution to such rho problem is to properly include the backreaction of the D3 brane on the warp factor. In the context of the Hamiltonian formalism we discussed above, this translates into the inclusion of the δ-function term in the constraint equation (38). In other words, the D3 brane position Y^i now also becomes a modulus of the bulk metric. The correction to the warp factor e^{-4A} can be determined through the linearized equation of motion for the RR four-form, perturbed by localized source [14, 27]. The resultant warp factor now becomes:

$$e^{-4A(y,c,Y)} = c + e^{-4A_0(y)} + \frac{\gamma}{2}k(Y,\overline{Y}) + [\text{hol.} + \text{antihol.}]. \tag{58}$$

Here the additional holomorphic and anti-holomorphic pieces satisfy the Laplace equation and contains pieces that generate the $\delta(y - Y)$, and also relevant to make $k(Y,\bar{Y})$ Kahler

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[10]We are grateful to Juan Maldacena for discussing this point with us.

invariant. With this understanding, one can show that (58) is consistent with the modified constraint (38). Upon substituting (58) instead of (49) into (57), one can now see that the integral becomes:

$$\int_{\mathcal{C}_4} d^4 y \sqrt{\hat{G}_4} e^{-4A(y,c,Y)} = V_4 \left(c + \frac{\gamma}{2} k(Y, \overline{Y}) \right) + V_4^W + [\text{hol.} + \text{antihol.}]$$
$$= \frac{1}{2} (\rho + \overline{\rho}) V_4 + V_4^W + [\text{hol.} + \text{antihol.}] . \tag{59}$$

In other words the D7 gauge coupling is now the real part of a holomorphic function, as demanded by supersymmetry. We therefore resolved the rho problem in the general warped background. This argument should be consistent with the field redefinition made in (56). It suggests the effective warp factor in the closed string sector after taking into account the backreaction of open strings must be accompanied by the shift $\frac{\gamma}{2} k(Y, \bar{Y})$ as in eq. (58)[11].

4 Discussions

In this paper, we developed a general approach to compute Kähler potentials involving simultaneously open and closed string moduli in warped compactification. As an illustration, we considered the position moduli of a D3-brane coupled to the universal Kähler modulus of a warped background and computed the combined Kähler potential. By restricting our results to only the closed string fluctuations, we also clarified some previous closed string derivations in the literature, in particular, the subtleties with gauges.

There are several potential applications of this work, one of which is a precise determination of the inflaton potential for D-brane inflation [12, 13, 32, 33]. As is well known, if the inflaton potential is generated by an F-term in supergravity, inflationary physics which is determined by the slow-roll parameter η is sensitive even to dimension six Planck suppressed operators. Therefore, the warped Kähler potential for the D3-brane moduli we obtained can play a crucial role in determining the eta parameter for this broad class of models. The combined Kähler potential is also an essential piece of information for determining the D3-brane vacua [34, 35] in strongly warped backgrounds.

We should point out that although the convenient gauge choice we suggested is useful and simple for extracting the kinetic terms for moduli that do not develop a classical potential, complications can arise for fields that appear into the classical potential. Examples include complex structure moduli in flux compactification. This is because the compensator $K \ddot{u} dt^2$ contributes to $\sqrt{g_{tt}}$ which multiplies the potential in the action. Upon integrating $\int d^{10}x \sqrt{-g_{10}} \, V(u)$ by parts, the compensator field K gives rise to an additional contribution $\sim K V'(u) \dot{u}^2$ to the kinetic terms. In fact, a naive extrapolation of our approach to the complex structure modulus of the conifold without taking this subtlety into account leads to a Kähler metric which is not positive definite, though on dimensional ground, one would expect the same parametric dependence as the expressions presented in [36, 1].

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[11]Note that one cannot substitute this backreacted warp factor into the DBI action of the D3-brane due to the well known self energy problem.

Another subtlety is that the potential term explicitly breaks the assumption that (arbitrary) constant u solves the background (zero-th) order equations of motion. Thus, there is less sense to talk about the linearlized equations of motion around the background if we are away from the vacua. To determine the non-supersymmetric vacua, we typically need to compute the Kähler potential. The linearized equations of motion and the Hamiltonian formulation, however, require that we should expand around the (yet-to-be-determined) vacua. It appears to be a chicken and egg problem, and approximate iterative or bootstrap approach might be necessary. The applicability of the Hamiltonian formulation to obtain kinetic terms away from the vacua remains open.

It would be interesting to derive explicitly Kähler potentials for moduli of this type. We leave this and related problems for future work.

Acknowledgements

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